Worked Solutions

Edexcel C3 Paper H

1. $x^2 - y^2 = (x - y)(x + y)$

Assume both x and y are even then $x - y = 2m$ where m is an integer.

$x + y = 2n$ where n is an integer.

$\therefore\ x^2 - y^2 = 4mn \neq 18$

Assume both x and y are odd then $x - y = 2m'$

$x + y = 2n'$

$\therefore\ x^2 - y^2 = 4m'n' \neq 18$

Assume x odd, y even then $x - y = 2m_1 + 1$ odd

$x + y = 2m_2 + 1$ odd

$\therefore\ x^2 - y^2 = 4m_1m_2 + 2(m_1 + m_2) + 1$ which is odd $\therefore$ contradiction. **(6)**

2. $\dfrac{\cancel{(2x-1)}\cancel{(x+3)}}{\cancel{(2x-1)}\cancel{(2x+1)}} \times \dfrac{x\cancel{(2x+1)}}{\cancel{(x+3)}(x+5)} = \dfrac{x}{x+5}$ **(6)**

3. (a) $y = 2x - 1$ $\therefore 2x = y + 1$

$\text{f}^{-1}: x \mapsto \dfrac{x+1}{2}, \quad x \epsilon \mathbb{R}$ **(2)**

(b) $\text{gf}^{-1}(x) = 3\left(\dfrac{x+1}{2}\right)^2 + 1$ **(2)**

(c) $3\left(\dfrac{x+1}{2}\right)^2 + 1 = \dfrac{7}{4}$

$3(x+1)^2 + 4 = 7$

$(x+1)^2 = 1$

$x = 0$ or -2 **(3)**

4. (a) x must be obtuse.

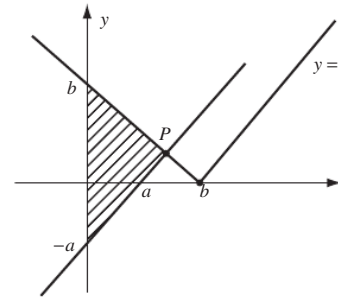$\therefore\ \tan x < 0, \sin x > 0$

$\cos^2 x = \dfrac{25}{169}$

$\tan^2 x = \sec^2 x - 1 = \dfrac{169}{25} - 1 = \dfrac{144}{25}$

$\tan x = -\dfrac{12}{5}$

(b) $\operatorname{cosec} 2x = \dfrac{1}{2\sin x \cos x}$

$= \dfrac{1}{2 \cdot \frac{12}{13} \cdot \frac{-5}{13}} = \dfrac{-169}{120}$

5. (a)

(b) at P $x - a = -(x - b)$

$x - a = -x + b$

$x = \dfrac{a+b}{2}$

$\text{Area} = \dfrac{1}{2}(b + a)\left(\dfrac{a+b}{2}\right) = \dfrac{(a+b)^2}{4}$

6. $y = \ln(3 - x)$

$y = \ln(x + 2) - 2$

$\ln(3 - x) = \ln(x + 2) - 2$

$\ln(x + 2) - \ln(3 - x) = 2$

$\dfrac{x + 2}{3 - x} = e^2$

$x + 2 = 3e^2 - xe^2$

$x(1 + e^2) = 3e^2 - 2$

$x = 2.40 \quad (2 \text{ d.p.})$

$y = -0.52 \quad (2 \text{ d.p.})$ **(8)**

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7. (a) (i) $x \cdot \dfrac{3}{3x + 4} + \ln(3x + 4)$ **(3)**

(ii) $5 \left(3x^2 + 7\right)^4 \cdot 6x$

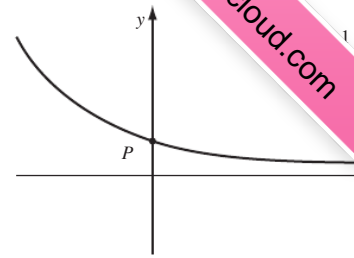$30x \left(3x^2 + 7\right)^4$ **(3)**

(b) $\dfrac{d}{dx} \left(\dfrac{\cos x}{\sin x}\right)$

$= \dfrac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$

$= \dfrac{-1}{\sin^2 x} = -\operatorname{cosec}^2 x$ **(4)**

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8. (a)

(b) $\dfrac{dy}{dx} = -\dfrac{1}{2}e^{-\frac{1}{2}x}$ at $x = q$: $\quad \dfrac{dy}{dx} = -\dfrac{1}{2}e^{-\frac{1}{2}q}$

gradient of normal $= 2e^{\frac{1}{2}q}$

Equation of normal is $y = 2e^{\frac{1}{2}q}x$

pt. Q lies on curve $\therefore e^{-\frac{1}{2}q} = 2e^{\frac{1}{2}q}q$

$1 = 2e^q q$

$\therefore q$ satisfies $2e^x x - 1 = 0$

$2e^x x = 1$

$x = \dfrac{1}{2e^x}$

$x_1 = 0.33516, x_2 = 0.35761, x_3 = 0.34967, x_4 = 0.$

$q = 0.352 \ (3 \text{ d.p.})$

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9. (*a*) (i) $R^2 = 9^2 + 40^2$

$R = 41$

$\tan \alpha = \dfrac{9}{40}$

$\alpha = 12.68$

(ii) $41 \cos(\theta + 12.68) = 4$ **(4)**

$\theta + 12.68 = 84.40$

$\theta = 71.7(1 \text{ d.p.})$ **(3)**

(*b*) $6 \sin^2 \theta - \sin \theta - 2 = 0$

$(2 \sin \theta + 1)(3 \sin \theta - 2) = 0$

$\sin \theta = -\dfrac{1}{2}, \text{ no solutions}$

$\sin \theta = \dfrac{2}{3}, \quad \theta = 41.8\,°, 138.2°$ **(5)**

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